[07-09-09B-T10]

Absolute value and linear inequalities

Providing k is a positive number and $a \neq 0$, the following theorems hold for all real numbers a and b.

- **Theorem 1** $\quad |x| < k \Longleftrightarrow -k < x < k$
- **Theorem 2** $\quad |x| > k \Longleftrightarrow x < -k$ or $x > k$
- **Theorem 3** $\quad |ax+b| < k \Longleftrightarrow -k < ax+b < k$
- **Theorem 4** $\quad |ax+b| > k \Longleftrightarrow ax+b < -k$ or $ax+b > k$

- **[EX1] Solve for x when $|x| \geq -7$.**

Solution.

Since the absolute value of a number is always positive, the absolute value of any number is always greater than any negative number. Therefore, every real number is a solution of $|x| \geq -7$; i.e. $x \in \mathbb{R}$.

- **[EX2] Solve for x when $|x| < -1$.**

Solution.

Since the absolute value of a number is always positive, there is no real number whose absolute value is less than a negative number. Therefore, $|x| < -1$ has solution set $\emptyset$.

- **[EX3] Solve for x when $|3x-8| \leq 5$.**

Solution.

Using theorem 2,

$|3x-8| \leq 5$

$\Longrightarrow -5 \leq 3x-8 \leq 5$

$\Longrightarrow 3 \leq x \leq 13$

$\therefore\ x \in [3,\ 13]$

■ **[EX4] Solve for x when $|3x-8| > 5$.**

Solution.

Using theorem 4,

$|3x-8| > 5$

$\Longrightarrow 3x-8 < -5$ or $3x-8 > 5$

$\Longrightarrow x < 1$ or $x > \frac{13}{3}$

$\therefore\ x \in (-\infty, 1) \bigcup (\frac{13}{3}, \infty)$

■ **[EX5] Solve for x when $|5(x+2)-13| < 15$.**

Solution.

Using theorem 2,

$|5(x+2)-13| < 15$

$\Longrightarrow |(5x-3)| < 15$

$\Longrightarrow -15 < 5x-3 < 15$

$\Longrightarrow \frac{-12}{5} < x < \frac{18}{5}$

$\therefore\ x \in (\frac{-12}{5}, \frac{18}{5})$